

Exploration Activity, Long Run Decisions, and the Risk Premium in Energy Futures*

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Abstract

We present evidence that the aggregate capital stock of firms in oil and gas exploration and development (E&D) as well as firms' inventories help in explaining the dynamics of the slope of the futures curve for crude oil. Standard structural approaches for modeling the futures curve either highlight the role of inventory (storage models) or the rate of extraction (production models), but both decisions are seldom modeled simultaneously. Here we build a new equilibrium model that has both features, and in addition, models the process of E&D capital accumulation, which can affect the cost of extraction as the oil industry drills in increasingly expensive fields. We show how the three decisions interact in a world of exhaustible resources. In a nutshell, a steeper futures slope not only increases the attractiveness of carrying inventory, but also provides greater value to accumulating E&D capital. Our model sheds light on the role of exhaustibility of resources on the increasing trend of real oil prices and capital accumulation, and the peaking of consumption. It also helps understand why inventories and E&D capital each

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negatively predict returns on oil futures, and is thus able to shed light on the negative relation between the slope and risk premium on oil futures.

Introduction

Recent years have seen the development of increasingly sophisticated technologies for the extraction of natural resources from costlier fields.¹ These new technologies brought to fruition by investments by the resource extraction industry have changed the current and expected future prices of resources and have important consequences for energy self sufficiency and stability of growth for North America. In this paper we ask if the investment in exploration and development (E&D) of resources has an impact or is affected by the keenly watched market statistics of current and future prices of the resource.

Of the most widely watched statistics in the futures market is the weak basis, which is the discounted value of the futures price less the spot price of the resource. We work with a closely related statistic, the relative basis, which is the weak basis divided by the current spot price of the resource. When this quantity is positive (negative) we say the futures market is in weak contango (backwardation). Of interest to practitioners and researchers is the economic information that determines the relative basis. The theory of storage (Kaldor (1939) and Working (1948)) implies that the futures relative basis is strongly positively related to inventories. We call this the “short-run” information about resource prices in the futures relative basis. However, as we will see below, inventory data, though very useful, is unable to explain the basis in certain periods. In addition in this paper, we argue that the futures relative basis also contains “long-run” information about resource prices, which has important implications for decisions such as the exploration and development of the resource extraction process. In particular, we will develop four stylized properties of oil futures prices that arise from the long-run risks faced by energy producers.

Stylized Fact 1: The aggregate capital stock of firms in E&D as well as firms’ inventories help in explaining the dynamics of the slope of the futures curve for crude oil.

The top left panel of Figure 1 shows the seasonally adjusted futures basis of crude oil. As can be seen the futures curve has mostly been in contango for the period from 2008 to 2013, while backwardation was more frequently prevalent earlier. The bottom left and right panels

¹In Alberta, Canada, new techniques have been developed to extract crude oil from bitumen using less water and energy and damage to the environment than previously envisaged. In the United States, new hydraulic fracturing technology has made the oil and gas trapped in previously inaccessible shale rock now economically feasible to extract.

Table 1: What Explains the Futures Weak Relative Basis For Crude Oil (1986:7 - 2014:9)?

No.	α	β_1	β_2	R^2
1	-0.933 [-7.402]	0.850 [6.837]		0.284
2	-0.149 [0.0215]		0.866 [4.796]	0.246
3	-0.821 [6.704]	0.678 [5.459]	0.611 [3.159]	0.401

We report the coefficients of the fitted monthly regression:

$$\text{Relative Basis}(t) = \alpha + \beta_1 \text{Inventory}(t-1) + \beta_2 \text{Capital Stock/GDP}(t-1) + \epsilon(t).$$

The weak relative basis on 1-year contracts in quarter t is $[e^{-r(t)}F(t) - S(t)]/S(t)$, where $F(t)$ is the 1-year futures prices at the beginning of each quarter and $S(t)$ is the spot price of WTI oil in Cushing, Oklahoma. Seasonal adjustment is done using the X-12 procedure (used by the US Department of Commerce). The explanatory variable “Inventory” stands for the seasonally adjusted total US stock of crude oil and petroleum products (in billions of barrels) excluding special purpose reserves at the end of each month. The “Capital Stock” is the sum of the “Property Plant and Equipment” variable in Compustat of firms in oil and gas field exploration services (SIC code 1382). T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation.

show seasonally adjusted inventory and the capital stock to GDP ratios, respectively. While inventory has been higher in recent years as well relative to the earlier part of the sample, the growth of the capital stock of firms in E&D firms has been far more spectacular. The ratio of capital stock to GDP was quite stable in our sample from 1986 to around 2001, but has grown very rapidly since then. This has been the period of rapid development of the shale oil plays in the US.

Table 1 reports simple linear regressions at a monthly frequency of the futures relative basis on inventory and the capital stock of E&D firms as a share of U.S. GDP (see the data appendix for sources of data).² As can be seen, while one quarter lagged inventory explains about 28 percent of the variation in the basis, the lagged capital to GDP ratio explains about

²Throughout this paper we look at the statistics of the one-year futures relative basis. While it would be of interest to study longer maturity futures, we are constrained by the lack of long historical times series on these longer term contracts. Two-year contracts started trading actively in mid 1990 and four-year contracts only in 1997. The correlations of the relative basis of the two-year and four-year contracts with the one-year contract are 99.7 and 98.7 over the subsamples, respectively.

25% of the variation in the relative basis (lines 1 and 2). Both variables have positive beta coefficients. When both variables are considered, we explain about 40% of the variation in the relative basis, and each variable remains significant. This suggests that both short and long run decision by firms are important determinants of the futures relative basis. The periods when the discounted futures price is higher than the spot price, inventory accumulates. In addition, firms raise more capital for E&D expenses in response perhaps to higher futures prices.

Table 2: Risk Premium on Crude Oil Futures at Alternative Horizons (1986:7 - 2014:9)

h	α	β_1	β_2	R^2
1	0.101 [0.900]	-0.073 [-0.509]	-1.113 [-1.758]	0.024
3	0.231 [0.821]	-0.155 [-0.454]	-0.322 [-1.472]	0.037
12	-0.501 [0.579]	0.795 [0.941]	-1.688 [-2.336]	0.065

We report the coefficients of the fitted monthly regression:

$$\sum_{i=0}^h \text{return}(t-1+i) = \alpha + \beta_1 \text{Inventory}(t-2) + \beta_2 \text{Capital Stock/GDP}(t-2) + \epsilon(t),$$

for $h = 1, 3,$ and 12 months. $\text{return}(t)$ is the return on purchasing a 2-month maturity futures contract at the beginning of the month $t - 1$ and closing it at the spot price, which is the 1-month maturity futures contract at t . The explanatory variable “Inventory” stands for the total US stock of crude oil and petroleum products (in billions of barrels) excluding special purpose reserves at the end of the month. The “Capital Stock” is the sum of the “Property Plant and Equipment” variable in Compustat of firms in oil and gas field exploration services (SIC code 1382). T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation.

Stylized Fact 2: The risk premium on crude oil futures is negatively related to both inventory and capital of E&D firms.

As seen in Table 2, the two variables explain 2.4 percent of the excess return variation at a monthly frequency, with each variable having negative coefficients. At longer horizons of 3 months and 12 month rolling returns (holding a sequence of 2-month contracts for 1 month) are predicted with R^2 of 3.7 percent and 6.5 percent, respectively.

Stylized Fact 3: As seen in the top panel of Figure 2, the aggregate consumption of petroleum products trended upwards from 1986 to 2007, but has fallen off since then. The bottom panel shows that the real spot price of oil has trended up around some fluctuations in the entire sample from 1986 to 2014. Therefore, despite increasing capital expenditures and inventories, consumption appears to have at least a local peak amid rising prices.

Stylized Fact 4: It is natural to think of long-run risk as pertaining to events that happen less frequently. The left panel of Figure 3 shows the variance frequency decomposition of the real spot price of oil. As seen in the figure, there is a large amount of variance of the weak relative basis that is of low frequency movements (every 6 years or less often). There is also a large proportion of high frequency movements (every 2 months or more often). Overall, the frequency decomposition is U-shaped, with these two extreme frequency movements explaining more of the variation in the relative basis than intermediate frequency movements. This is to be contrasted with the analogous decomposition for most macroeconomic series such as industrial production, which show a spike at a frequency such as 2.5 years (see e.g. Figure 6.5 in Hamilton (1994)). The lack of such a frequency suggests that the commodity pricing cycle is distinct from the business cycle, but we will examine this more in detail in this paper.

In this paper, we build a model of the long run decision making of resource producing firms that in equilibrium will lead to firms' decisions and spot and futures prices having these stylized properties. We model demand shocks that drive the business cycle fluctuations in oil prices, but also build in the implications of the exhaustibility of the total resource base, and firms' decisions on capital accumulation that manage their extraction costs as they extract from increasingly costly fields. In addition, firms' choose inventories to smooth fluctuations in demand and extraction.

We start with a 2-period version of the model for which we can provide closed form expressions for the value of extraction options and can characterize the optimal investment and inventory policy quite tractably. The closed-form expressions for the futures basis and risk premium show explicitly the role of the real decisions variables in determining these variables. With comparative static exercises we show how the relationship between the decision and financial variables varies with the level of the capital stock. In particular, we highlight the aggressiveness of extraction decisions and inventory choices and their role in affecting the

financial variables. The comparative static results are at least qualitatively similar to the first two stylized facts.

To study the dynamics of these variables and their relationship to the data, we next build an infinite horizon model. In dynamic models with inventory restricted to be positive, it is not possible to obtain closed-forms as has been pointed out by several authors. So, we solve the model using projection methods as has been popularized by the work of Judd (1999). Simulations of the model show that both inventory and capital accumulation impact the basis and the risk premium with the same signs as in the data. The model also shows that even though consumption peaks, and real prices of the resource trend upwards, the cycles of inventory and capital accumulation are quite stable, as increasingly costlier resources are extracted. While the resource is not completely exhausted even after more than a hundred years, consumption drops significantly after about half the resource base is exhausted, while capital fluctuates with the demand cycle. Therefore, the first two stylized facts are consistent with a model of near exhaustion of the resource or the peak consumption view. The model also generates a frequency decomposition of spot prices similar to that in the data. The model R^2 for the basis is not as high as in the data, and it is slightly higher than the data in explaining the risk premium.

Our model contributes to the literature on resource extraction and storage. Most existing models have either one of these features. Models of storage assume exogenous extraction decisions (e.g. Deaton and Laroque (1992) and Routledge, Seppi, and Spatt (2000)). Starting with the seminal work of Hotelling (1931), models of exhaustible resource extraction on the other hand, allow no storage (e.g. Pindyck (1980), Litzenberger and Rabinowitz (1995), Carlson, Khokher, and Titman (2007) and Kogan, Livdan, and Yaron (2009)). The same is true of models that allow production of commodities (see e.g. Casassus, Collin-Dufresne, and Routledge (2008)). In the context of agricultural commodities, there is an older literature that has production and storage in equilibrium, but the analysis in such models does not apply to exhaustible resources, where equilibrium profits are compatible with competitive equilibrium due to limitations in supply (e.g. Scheinkman and Schechtman (1983)). With exhaustible resources, as pointed out in Litzenberger and Rabinowitz (1995), there are profits at the time of extraction that are optimized by the extraction timing decision of resource firms, a feature that we model explicitly. The model sheds light on the negative relation between the futures

basis and the risk premium as has been pointed out by several authors (e.g. Fama and French (1987), Gorton and Rouwenhourst (2006), Erb and Harvey (2006), and Baker and Routledge (2012)).

The paper structure is as follows. In Section 1, we formulate the 2-period model of optimal resource extraction and the relative basis, and study its comparative statics. In Section 3, we extend our analysis to the infinite horizon model and study the dynamic properties of the model using simulations. Section 4 concludes. Three appendices contain the data description and sources, some analytically results, and the numerical procedure for solving the infinite horizon model, respectively.

1 A Simple Two Period Model of Resource Extraction and Exploration Activity

We build on the two period version of the model of LR, with several generalizations. The most significant addition is of an E&D (investment) decision that reduces costs of future extraction of the resources. To tractably analyze the investment decision with technology spillover, we introduce multi-plant firms. Assume a continuum of price taking identical resource production multi-plant firms, each of which owns an equal share of reserves. We will focus our analysis on the representative firm.

We start with a description of the demand side of the model. The demand function for the resource at time t is given by simple function $q_t = f(S_t, \epsilon_t)$, where ϵ_t is a demand shock realization for the resource at date t . Without loss of generality, we set $\epsilon_0 = 0$, and $\epsilon_1 = \epsilon$. Conditional on a realization of ϵ , the inverse demand function is $s = f^{-1}(q_t; \epsilon_t)$.

Supply of the resource is optimally determined by the firm. The resource can be extracted from wells of varying quality, which is parsimoniously captured by a variable x . Wells x are uniformly distributed $x \in [0, \bar{x}]$ in period 0. Well x is operated by plant x owned by the firm and has access to technology with extraction cost in period 0 of $x g(K_0)$, where K_0 is the amount of capital in the industry (to be discussed below). Let R_0 be the reserves available at date 0. At date 0, the plant level decisions determine the cutoff reserve quality (extensive margin), x_0^e .

As discussed in the introduction, there are interesting relationships between the slope of the futures curve, real decisions, and expected returns on futures strategies. To address these, our model must build in the price of risk in energy commodities. Following a long literature in asset pricing, we specify an exogenous pricing kernel with a constant price of risk of the form:

$$M_1 = M_0 \cdot \exp(-r - \sigma_M \epsilon). \quad (1)$$

To keep things simple, we have specified that the kernel depends on the shock to energy consumption, so that marginal utility is high in periods of low energy demand. While oil and total consumption are not perfectly correlated in real data, we could generalize this assumption with an increase in computational complexity by having a second not perfectly correlated shock to the kernel. Using the kernel, we can compute all expectations under the risk-neutral measure, which we will denote as $E^Q[\cdot]$.

We assume that all investments in technology are made at the firm level, and affects extraction costs for resources of all grades. In particular, extraction cost for well x at time t is given by $x g(K_t)$, where K_t is the amount of capital in E&D. The timing of capital installation is as follows: At date 0, the firm inherits capital of K_0 from past decisions. The firm can augment this capital stock by incurring E&D expenses, which we call investing. The new capital will follow the standard process

$$K_1 = (1 - \delta) K_0 + I_0. \quad (2)$$

The investment choice is made *before* before any extraction decisions are made. Conditional on the investment choice at the firm level, each plant chooses its extraction decision to maximize the profits of the plant. Conditional on the firm level investment, the plant level maximization can be written as:

$$\pi_0^x = \max_{0 \leq Q_0^x \leq \frac{R_0}{\bar{x}}} S_0 Q_0^x - Q_0^x x g(K_0) + e^{-r} E^Q[(S_1 - x g(K_1))^+] \left(\frac{R_0}{\bar{x}} - Q_0^x \right). \quad (3)$$

In particular, for a firm with positive and interior production

$$S_0 - x g(K_0) = C(x g(K_1)), \quad (4)$$

where $C(x g(K_1))$ is the value of a 1-period call option with exercise price of $x g(K_1)$. The left-hand side is the net gain to current extraction, while the right-hand side is the value of delaying extraction. It is useful to note at this point that the call option valuation in (4) is quite similar to a regular American option, with the only difference being that the price at each date of the resource is determined by the aggregate optimal extraction decision of all producers using the inverse demand function. The Kuhn-Tucker optimality condition at the boundaries for the extraction choice of plant x satisfy

$$[S_0 - x g(K_0) - C(x g(K_1))] Q_0^x = 0 \quad \text{or} \quad [S_0 - x g(K_0) - C(x g(K_1))] \left(\frac{R_0}{\bar{x}} - Q_0^x \right) = 0. \quad (5)$$

We complete the analysis of the model by determining the investment choice at date 0 in the context of the model without and with storage in the following subsections.

1.1 Model Without Storage

We now show how the cutoff resource quality (the extensive margin is determined) x_0^e . At date 1 since there are no further options and no inventory, all plants with available resource and extraction costs smaller than the price ($x g(K_1) < S_1$) will extract. Given installed capital of K_1 , therefore, aggregate production at date 1 will be

$$Q_1(x^e, \epsilon) = \left(\int_{x_0^e}^{S_1/g(K_1)} \frac{1}{\bar{x}} dx \right) R_0 = \frac{S_1/g(K_1) - x_0^e}{\bar{x}} R_0. \quad (6)$$

Hence, the date 1 price is $\tilde{S}_1 = s(Q_1(x^e, \epsilon); \epsilon)$. Let $C(x|x_0^e, K_0, I_0)$ be the value of the extraction call option for the firm when the extensive margin is x_0^e , K_0 is capital at date 0, and I_0 is investment of at date 0. Then x_0^e satisfies the fixed-point condition:

$$S_0(x_0^e) - x_0^e g(K_0) = C(x_0^e g(K_1)|x_0^e, K_0, I_0), \quad (7)$$

when it lies in the interior of the interval $[0, \bar{x}]$, and with the boundary conditions:

$$x_0^e(K_0, I_0) = 0 \text{ if } s(0) g(K_0) < C(0|0, K_0, I_0), \quad (8)$$

$$= \bar{x} \text{ if } s(\bar{x}) g(K_0) - \bar{x} > C(\bar{x}|\bar{x}, K_0, I_0). \quad (9)$$

The firm maximizes total profit at date 0

$$\begin{aligned} \pi_0 &= \max_{I_0 > 0} E^Q \left[\int_0^{\bar{x}} \pi_0^x dx - P_0 I_0 \right] \\ &= \max_{I_0 > 0} \left[S_0 \frac{x_0^e}{\bar{x}} - \frac{(x_0^e)^2}{\bar{x}} g(K_0) \right] R_0 + \left(\int_{x_0^e}^{\bar{x}} C(x g(K_1)|x_0^e, K_0, I_0) dx \right) \frac{R_0}{\bar{x}} - P_0 I_0 \end{aligned} \quad (10)$$

where P_0 is the price of capital at date 0 in consumption goods at that date. To compute expected profit we calculate the maximal investment choice numerically by choosing over a grid of values.

We now make specific assumptions on the demand function and the distribution of shocks that enable us to solve for the firm value in closed form. Specifically, we assume a linear demand function in each period of the form: $q_0 = a - b S_0$, and $q_1 = a \cdot e^{\mu + \sigma \epsilon} - b S_1$, where the demand shock ϵ is distributed $N(0, 1)$. The assumption implies resource expected demand growth at the rate μ . Now using the demand function at date 1, equilibrium entails that:

$$\frac{1}{\bar{x}} (S_1/g(K_1) - x_0^e) R_0 = a e^{\mu + \sigma \epsilon} - b S_1.$$

Solving for S_1 we have

$$S_1 = \frac{a e^{\mu + \sigma \epsilon} + \frac{x_0^e}{\bar{x}} R_0}{b + \frac{R_0}{\bar{x} g(K_1)}}. \quad (11)$$

In addition, we have

$$S_0 = 1/b \left(a - \frac{x_0^e}{\bar{x}} R_0 \right). \quad (12)$$

Since the resource prices at each date are dependent on the extraction choices of firms, which in turn depends on capital inherited at date 0, and their investment choice, we first formulate the value of the extraction option conditional on both these variables.

Proposition 1 *The value of the extraction call option at date 0, given installed capital K_0 , investment I_0 , and cut-off resource quality $x_0^e \in [0, \bar{x}]$ for plant x is*

$$\begin{aligned}
C(x g(K_1)|x_0^e, K_0, I_0) &= \frac{a e^{-r}}{D} \left[e^{(\mu - \sigma_M \sigma + 0.5 \sigma^2)} N(-d_1) - k N(-d_2) \right], \\
d_1 &= \frac{\log(k) - \mu + \sigma_M \sigma - \sigma^2}{\sigma}; & d_2 &= \frac{\log(k) - \mu + \sigma_M \sigma}{\sigma}; \\
k &= \frac{1}{a} \left(D x g(K_1) - \frac{x_0^e}{\bar{x}} R_0 \right); \\
D &= b + \frac{R_0}{\bar{x} g(K_1)}.
\end{aligned}$$

The value of a put option is

$$P(x|x^e, K_1) = \frac{a e^{-r}}{D} \left[k N(d_2) - e^{(\mu - \sigma_M \sigma + 0.5 \sigma^2)} N(d_1) \right].$$

The proof is in the appendix.

Using the stock price in (11) implies that the forward price for the linear demand case satisfies:

$$F_0 = E^Q[s(Q_1; \epsilon)] = \frac{a e^{\mu - \sigma_M \sigma + 0.5 \sigma^2} + \frac{x_0^e}{\bar{x}} R_0}{b + \frac{R_0}{\bar{x} g(K_1)}}. \quad (13)$$

What does this simple two-period model imply about the relationship between investment and futures basis? It is hard to sign this relationship in general, we can for given extraction x_0^e decisions. In this case, as seen above, the futures price is increasing in extraction costs, while the spot price, conditional on x_0^e does not depend on it. Therefore, the futures basis is increasing in $g(K_1)$. An increase in the extraction costs implies a lower expected supply in the future, so that prices will be higher in the future, Under the assumption that $g'(K_1) < 0$, we will have a negative relationship between capital and the futures basis, which is counterfactual. However, if x_0^e is higher in periods of high capital, which is reasonable, since the firm is likely to be more aggressive with its extraction policy in periods of low future extraction costs, the relationship can well turn positive. We will look at this relationship further in Section 1.3 below.

1.2 Model With Storage

As mentioned in the introduction, existing models of resource extraction do not allow for storage, while models with inventory do not have optimal resource extraction. In addition, none of these models have exploration activity. Here we provide the analysis of a model with production, storage and exploration. The model will help us address the stylized facts noted in the introduction on the positive comovement of exploration activity, extraction, and inventory accumulation.

We continue to formulate the decisions of the multi-plant firm in the subsection 1.1 assuming once again that E&D investment decisions are made before extraction and inventory decisions. We assume that investment and inventory decisions are made at the firm level, while extraction decisions are made at the plant level. Essentially, in the model with storage, the firm has two substitutable ways of providing resource to customers at date 1: it can either defer date 0 extraction and extract in date 1, or it can extract in date 0, and carry inventory to date 1. Which strategy is more profitable? Each has its own advantages, and the tradeoff is to a large part determined by storage costs and the expected change in extraction costs. If the latter are expected to increase rapidly, for example, it might be worthwhile for the firm to extract in date 0 and carry inventory. In addition, the price protection offered by holding the resource in the ground (as in the case of no storage) implies that an increase in uncertainty will make the delayed extraction choice more profitable.

The plant level optimization is very similar to the case without storage, albeit with different equilibrium resource prices. The objective function of the plant still satisfies (3) and its optimal extraction policy is determined as in (4). Given this, the profit at the firm level is

$$\begin{aligned} \pi_0 = & \max_{I_0 > 0} \max_{x_0^e \in [0, \bar{x}]} \max_{Z_1 \in [0, \frac{x_0^e}{\bar{x}} R_0 + Z_0]} S_0 \left[\frac{x_0^e}{\bar{x}} R_0 + Z_0 - Z_1 \right] - 0.5 \frac{(x_0^e)^2}{\bar{x}} g(K_0) R_0 - P_0 I_0 \\ & + E^Q \left[e^{-(r+u)} \tilde{S}_1 Z_1 \right] + \left(\int_{x_0^e}^{\bar{x}} C(x g(K_1) | Y_0) dx \right) \frac{R_0}{\bar{x}}, \end{aligned} \quad (14)$$

where Y_0 denotes the vector of state variables: $Y_0 = (x_0^e, K_0, I_0, Z_0, Z_1)$. Conditional on extraction, investment, and storage policy, the optimality conditions for the extraction policy are similar to the case without storage, but now building in the impact of storage on prices at

both dates:

$$S_0 - x_0^e g(K_0) = C(x_0^e e^{g(K_1)} | Y_0), \text{ if } 0 < x_0^e < \bar{x}, \quad (15)$$

$$x_0^e = 0 \text{ if } s(0) g(K_0) < C(0 | Y_0), \quad (16)$$

$$x_0^e = \bar{x} \text{ if } s(\bar{x}) g(K_0) - \bar{x} > C(\bar{x} | Y_0), \quad (17)$$

where for parsimony we have written the date 0 price $s(x_0^e)$, only as a function of the choice of the extensive margin x_0^e , even though it depends on the entire vector Y_0 .

We can similarly formulate the firm's optimal storage policy conditional on the investment and extraction decisions. Given our assumption on the inverse demand function we can write the price at date 1 as

$$\tilde{S}_1 = s(Q_1 + Z_1 e^{-u}; \tilde{\epsilon}). \quad (18)$$

Continuing to assume that the firm is a price taker, the first order condition with respect to inventory, Z_1 is

$$-S_0 + e^{-(r+u)} E^Q[S_1] = 0 \text{ if } 0 < Z_1 < \frac{x^e}{\bar{x}} R_0 + Z_0, \quad (19)$$

$$< 0 \text{ if } Z_1 = 0, \quad (20)$$

$$> 0 \text{ if } Z_1 = \frac{x^e}{\bar{x}} R_0 + Z_0. \quad (21)$$

The interior case in (19) determines the regular textbook equation for the value of a forward contract, while (20) occurs in "stockouts", when all available resource is consumed, and hence no inventory is carried. Finally, (21) occurs in periods when nothing is consumed at date 0, and all produced resource is stored for future consumption. We will discuss explicitly below how Z_0 and x_0^e are determined. The investment policy is maximized numerically over a grid of values similar to the case without storage.

Specializing again to the linear demand case: $q_0 = a - b S_0$, and $q_1 = a \cdot e^{m+\sigma\epsilon} - b S_1$, enables us to solve for resource prices and extraction options in closed form. Equilibrium at date 1 now requires:

$$\frac{1}{\bar{x}} (S_1/g(K_1) - x_0^e) R_0 + Z_1 e^{-u} = a e^{m+\sigma\epsilon} - b S_1.$$

Solving for prices, we now have

$$S_0 = 1/b \left(a + Z_1 - Z_0 - \frac{x_0^e}{\bar{x}} R_0 \right), \quad (22)$$

$$S_1 = \frac{a e^{\mu+\sigma\epsilon} + \frac{x_0^e}{\bar{x}} R_0 - Z_1 e^{-u}}{b + \frac{R_0}{\bar{x}g(K_1)}}. \quad (23)$$

Similar to Proposition 1 we solve for the extraction option value in closed form conditional on all firm level decisions.

Proposition 2 *The value of the extraction call option at date 0 in the presence of a storage technology with proportional storage costs of u , given installed capital K_1 , cut-off resource quality $x_0^e \in [0, \bar{x}]$ and resource storage amount of Z_1 for a resource with current extraction cost of x is*

$$\begin{aligned} C(xg(K_1)|Y_0) &= \frac{a e^{-r}}{D} \left[e^{(\mu - \sigma_M \sigma + 0.5 \sigma^2)} N(-d_1^s) - k N(-d_2^s) \right], \\ d_1^s &= \frac{\log(k^s) - m - \sigma_M \sigma - \sigma^2}{\sigma}; \quad d_2^s = \frac{\log(k^s) - m - \sigma_M \sigma}{\sigma}; \\ k^s &= \frac{1}{a} \left(D x g(K_1) - \frac{x_0^e}{\bar{x}} R_0 + Z_1 e^{-u} \right); \\ D^s &= b + \frac{R_0}{\bar{x}g(K_1)}. \end{aligned}$$

The value of a put option is

$$P(x|Y_0) = \frac{a e^{-r}}{D^s} \left[k^s N(d_2^s) - e^{(\mu - \sigma_M \sigma + 0.5 \sigma^2)} N(d_1^s) \right].$$

The proof is similar to that of Proposition 1.

Using the stock price in (23) implies that the forward price for the linear demand case satisfies:

$$F_0 = E^Q[s(Q_1; \epsilon)] = \frac{a e^{\mu - \sigma_M \sigma + 0.5 \sigma^2} + \frac{x_0^e}{\bar{x}} R_0 - Z_1 e^{-u}}{b + \frac{R_0}{\bar{x}g(K_1)}}. \quad (24)$$

The partial relation between investment and the futures basis is essentially the same as for the case without storage. In addition, higher inventories, ceteris paribus, imply lower futures prices due to an increase in supply in period 1.

Combing the futures and expected spot price we have:

$$\frac{E[S_T] - F_0}{F_0} = \frac{a e^{\mu+0.5\sigma^2} - e^{\mu-\sigma_m\sigma+0.5\sigma^2}}{a e^{\mu-\sigma_m\sigma+0.5\sigma^2} + \frac{x_0^e}{\bar{x}} R_0 - Z_1 e^{-u}} \quad (25)$$

which is the expected return from the long futures position. This quantity is often called the “risk premium” (see e.g. Gorton and Rouwenhorst (2006)). Keynes observed that speculators mostly take the short side of futures contracts and therefore require a risk premium for holding the commodity risk. Therefore, the futures price that the producers (hedgers) would sell at should be lower than the expected spot price that they could obtain by holding the commodity and selling in the future. However, here we notice that the risk premium for the commodity is not only the standard $-\sigma_M \sigma$, but also depends on the firm’s investment policy through its effect on the firm’s production and inventories, each of which is endogenous and related to the firm’s investment policy.

We now provide a description on how the extensive margin and inventories are jointly determined in the storage version of the model. For a given choice of the extensive margin x , we use (19) - (21) and (22) - (23) to determine inventory as

$$\begin{aligned} Z_1(x_0^e) &= \frac{e^{r+u}(-a + \frac{x_0^e}{\bar{x}} R_0 + Z_0)(b g(K_1)\bar{x} + R_0) + b g(K_1)\bar{x}(a e^{\mu-\sigma_M\sigma+0.5\sigma^2} + \frac{x_0^e}{\bar{x}} R_0)}{e^{r+u}(b g(K_1)\bar{x} + R_0) + e^{-u} b g(K_1)\bar{x}} \\ &= 0 \text{ if } s(x_0^e|Z_1 = 0) > e^{-(r+u)} F(x_0^e|Z_1 = 0) \\ &= \frac{x}{\bar{x}} R_0 \text{ if } s(x_0^e|Z_1 = \frac{x_0^e}{\bar{x}} R_0 + Z_0) < e^{-(r+u)} F(x_0^e|Z_1 = \frac{x_0^e}{\bar{x}} R_0 + Z_0), \end{aligned}$$

for interior, and boundary choices, respectively. In particular, optimal inventory for the interior case is the solution to the equation $s(x_0^e) = e^{-(r+u)} F(x_0^e)$. We then use this optimal inventory function in the first order conditions for the extensive margin in (15) – (17).

One immediate implication of introducing storage possibilities into the model is that it makes the futures basis less variable. In particular, whenever an interior level of inventories is chosen, we have by construction that $S_0 = e^{-(r+u)} F$, so that the futures relative basis identically equals $(e^{-r} F - S_0)/S_0 = e^u - 1$. Departures from this constant slope therefore occur only when inventory is constrained, either from becoming negative or from it exceeding

the sum of incoming inventory and current output. In the former case, $S_0 > e^{-(r+u)} F$ leading to a backwarddated futures curve, while the latter case leads to a futures curve in contango.

1.3 Comparative Statics of Optimal Firm's Decisions, Futures Basis, and the Risk Premium with Respect to Mean Demand Shocks

As discussed above, it is hard to analytically determine the relation between the optimal firm's decisions and asset prices since the extensive margin is endogenous and affects prices. We consider the comparative statics for a specific numerical example in Figure 4 with respect to alternative levels of the mean of demand shocks, m . In particular, in this figure, we examine the extensive margin, investment, inventory, weak relative basis, and the risk premium for three levels of the period 0 capital stock. All optimal decisions are calculated for the model with storage in Section 1.2.

As seen in the first row, for each level of the capital stock, the extensive margin is flat or declines with higher demand shocks, as the resource is provided to consumers in periods when their demand is higher. In periods of high capital, there is greatest date 0 consumption, as the higher capital enables the firm to extract costlier resources at date 1. The results for investment in the second row are quite interesting since there are two effects: first, higher demand shocks lead to higher investment from a substitution effect, but higher demand shocks also imply that higher extraction costs (from low future capital) will still lead to profitable extraction leading to lower investment. Overall, the relation between investment and demand is negative for medium levels of capital, but positive for low and high levels.

The third row shows the optimal inventory decision. During periods of high capital, as seen, inventory is the highest for high levels of capital stock, because the firm takes the most aggressive consumption decision at such times, extracting the costliest resources and must have inventories for future consumption in case of adverse shocks, specially for low mean demand. Overall, for both medium and high levels of capital, the relation between inventory and mean demand is negative.

The fourth row shows that the relation between mean demand and the basis is quite different for alternative levels of capital. For low capital levels, the basis is increasing, while for high

capital it is decreasing. For high capital level, investment increases in mean demands, lowers future extraction costs leading to greater supply and lower futures prices. Conversely, for low capital, lack of future extraction implies higher future prices. For medium levels, it has a nonmonotonic relation, negative for low demand levels, and positive for high demand levels arising from the combination of the two effects. Finally, the fifth row, shows that the risk premium is lower for higher levels of capital stock due to the greater expected resources in the second period. In addition, the risk premium is higher for higher demand levels for each level of the capital stocks.

Thus, the relationships between investment and the financial variables are quite complex, and can change sign for different levels of the capital stock. For example, the relation between investment and the risk premium is negative for medium levels of the capital stock, and positive for high levels. The relationships between the capital stock and the financial variables are more stable, with a higher basis and lower risk premium for a higher capital stock. These relations have the same sign as what we observe empirically in the introduction. Still, these results are only comparative statics. In the next section, we will see if they hold dynamically in simulations of an infinite horizon model.

2 The Infinite Horizon Model with Production, Exploration, and Storage

We preserve much of the structure of the 2-period model. The one additional assumption that we make here is that there are adjustment costs to investment, an assumption that is standard in the investment literature to reduce the volatility of the investment process (see e.g. Kogan, Livdan, and Yaron (2009) in the context of a production model for commodities). This will help us provide a more empirically realistic model relationship between investment and the futures basis.

The demand function for the resource at time t is once again given by $q_t = f(S_t, \epsilon_t)$, where ϵ_t is a demand shock realization for the resource at date t . Conditional on a realization of ϵ_t , the inverse demand function is $s = f^{-1}(q_t; \epsilon_t)$. In addition, we assume the same form for the pricing kernel as for the 2-period model as specified in (1). We assume that the demand shock

follows a mean-reverting Ornstein-Uhlenbeck (OU) process:

$$\epsilon_{t+1} - \epsilon_t = -k_\epsilon \epsilon_t + \sigma_\epsilon (1 + |\epsilon_t|) e_t, \quad (26)$$

where $e_t \sim N(0, 1)$. The use of mean-reverting demand shocks is standard in the commodity pricing literature (e.g. Carlson, Khokher, and Titman (2007) and Pirrong (2012)). The process exhibits time varying volatility, which has a V-shaped relation with the demand shock. Therefore, volatility is high when demand is extremely low or extremely high. This feature captures the essence of time varying uncertainty of fundamentals that is now standard in macroeconomics and finance (see e.g. Bansal and Yaron (2004) and Bloom (2009)). We model this feature to potentially generate a time varying risk premium of the resource.

Let x_t^e be the extensive margin at the start of time t . The plant level decisions determine the increase in the extensive margin, i_t , so that $x_{t+1}^e = x_t^e + i_t$. Then at date t , the total production equals

$$Q_t = R_0 \cdot \int_{x_t^e}^{x_t^e + i_t} \frac{1}{\bar{x}} dx = R_0 \frac{i_t}{\bar{x}}. \quad (27)$$

The total extraction costs incurred by the firm at date t are

$$C_t = g(K_t) \cdot R_0 \cdot \int_{x_t^e}^{i_t + x_t^e} \frac{x}{\bar{x}} dx = \frac{1}{2} g(K_t) R_0 \frac{(x_t^e + i_t)^2 - (x_t^e)^2}{\bar{x}}. \quad (28)$$

It is useful to note that extraction costs are not simply proportional to i_t^2 , but instead are proportional to $i_t^2 + 2x_t i_t$. This is because an increase in the extensive margin leads to higher resource extraction costs as lower quality wells are accessed. An interesting implication is that the industry will have to maintain a higher level of capital stock over time to maintain a constant level of extraction costs.

We assume that the firm also has a costly storage technology. It is able to place a non-negative quantity Z_t in storage at time t . Storage costs are a proportion u of the quantity stored so an amount Z_t placed in storage at t , will make available an amount $Z_{t+1} = Z_t(1 - u)$ at $t + 1$. The firm behaves competitively in production markets, and we assume here that its storage decision has no price impact either. We will extend the analysis for the case of a non-negligible storage decision in future versions of the paper. For the competitive case,

we alternatively could assume that inventory decisions are made by a risk neutral speculator. However, with complete markets, the equilibrium will be identical with storage by either the firm or speculators. Combining production as in (27) and inventory, the total amount available for consumption in period t is

$$q_t = Q_t + Z_t - Z_{t+1}. \quad (29)$$

If there is a stockout, then $Z_{t+1} = 0$, that is, all available resource is consumed in period t .

To solve for equilibrium prices and quantities, we solve the related problem of a social planner who maximizes the discounted expected consumer plus producer surplus (see e.g. Weinstein and Zeckhauser (1975) and Carlson, Khokher, and Titman (2007)). The social surplus at time t is therefore,

$$SS_t = \int_0^{q_t} s(q; \epsilon_t) dq - C_t - P_t I_t, \quad (30)$$

$$= \int_0^{\frac{i_t}{\bar{x}} R_0 + Z_t - Z_{t+1}} s(q; \epsilon_t) dq - \frac{1}{2} g(K_t) R_0 \frac{(x_t^e + i_t)^2 - (x_t^e)^2}{\bar{x}} - P_t I_t, \quad (31)$$

where total production, costs of production, and consumption, are given in (27), (28), and (29), respectively, and P_t is the price of capital goods in units of consumption goods at date t . We hold $P_t = 1$ for all t .

The social planning problem can be solved by standard dynamic programming methods. The Hamilton-Jacobi-Bellman equation is

$$J(x_t^e, Z_t, K_t, \epsilon_t) = \max_{i_t \in [0, \bar{x} - x_t^e], Z_{t+1} \in [0, \frac{i_t}{\bar{x}} R_0 + Z_t], 0 \leq I_t \leq \bar{I}(K_t)} SS_t + e^{-r} E^Q [J(x_t^e + i_t, e^{-u} Z_{t+1}, e^{-\delta} K_t + I_t, \epsilon_{t+1})]. \quad (32)$$

Note that we have placed an upper bound on investment, potentially as a function of the capital stock, to capture the essence of adjustment costs. To economize on notation below, we will suppress the arguments of the J function and write $J_t = J(x_t^e, Z_t, K_t, \epsilon_t)$ and $J_{t+1} = J(x_t^e + i_t, e^{-u} Z_{t+1}, e^{-\delta} K_t + I_t, \epsilon_{t+1})$.

The first order conditions for this problem are:

$$\frac{R_0 (s(q_t \epsilon_t) - (x_t^e + i_t) g(K_t))}{\bar{x}} + e^{-r} E^Q[J_{x,t+1}] \leq 0; = 0 \text{ if } i_t > 0 \quad (33)$$

$$\frac{R_0 (s(q_t \epsilon_t) - (x_t^e + i_t) g(K_t))}{\bar{x}} + e^{-r} E^Q[J_{x,t+1}] \geq 0 \text{ if } i_t = \bar{x} - x_t^e \quad (34)$$

$$-s(q_t; \epsilon_t) + e^{-(r+u)} E^Q[J_{Z,t+1}] \leq 0; = 0 \text{ if } Z_{t+1} > 0, \quad (35)$$

$$-P_I + e^{-r} E^Q[J_{K,t+1}] \leq 0; = 0 \text{ if } 0 < I_t < \bar{I}(K_t), \quad (36)$$

$$-P_I + e^{-r} E^Q[J_{K,t+1}] \geq 0 \text{ if } I_t = \bar{I}(K_t), \quad (37)$$

It is worth noting that the optimality of the extensive margin and investment must be checked at both lower and upper boundaries. However, we only write the optimality condition for inventory at the lower boundary (zero). The optimality condition for inventory at the upper boundary (sum of production output and inventory carried over) will never be chosen if there is an Inada condition on the inverse demand function.

The first order conditions can be written as functions of the partial derivatives of the value function at date t rather than date $t + 1$ by use of the envelope theorem. For the extensive margin, it implies that

$$J_{x,t} = \frac{\partial}{\partial x_t} SS_t + e^{-r} E^Q[J_{x,t+1}] = -g(K_t) \frac{i_t}{\bar{x}} R_0 + e^{-r} E^Q[J_{x,t+1}]. \quad (38)$$

For incoming inventory, we simply have

$$J_{Z,t} = \frac{\partial}{\partial Z_t} SS_t = s(q_t, \epsilon_t), \quad (39)$$

which implies that $E^Q[J_{Z,t+1}] = E^Q[s(q_{t+1}, \epsilon_{t+1})]$. Finally for the capital stock,

$$J_{K,t} = -\frac{1}{2} g'(K_t) R_0 \frac{(x_t^e + i_t)^2 - (x_t^e)^2}{\bar{x}} + e^{-(r+\delta)} E^Q[J_{K,t+1}]. \quad (40)$$

We solve the HJB equation using projection methods as described in Judd (1999). Using the policy functions written in polynomial form, we can calculate expected future production in each state, and hence using the inverse demand function and the Markovian shocks, we can compute the forward prices as the expected value of the future spot price under the risk-neutral

measure. We note that all expectations are calculated using Gaussian Quadrature. Details of the approximation method are provided in Appendix 2.

3 Explaining the Stylized Facts

In this section we provide simulation results from the infinite horizon model in Section 2. Before doing so, we need to specify choices made on the demand function and the cost function in the model. We use an inverse demand function of the form: $s(q_t, \epsilon_t) = e^{b+\epsilon_t}/q_t^\alpha$, where $0 < \alpha < 1$ and ϵ_t follows the O-U process in (26). Such a demand function is standard in commodity pricing papers (see e.g. Carlson, Khokher, and Titman (2007) and Kogan, Livdan, and Yaron (2009)). The extraction cost function that we use is of the form $g(K) = \frac{\gamma}{K^p}$, where $p > 0$. This implies that extraction costs decline as capital accumulates, but explode as capital tends to zero, so that positive capital is required to ensure the supply of the resource.

We next discuss a set of results for some parameter values assumed for the model.

3.1 Parameter Values for Model

For the current version of the paper we choose the demand function parameter $\alpha = 0.5$, and $b = .1$. The latter parameter only governs the average level of demand. An $\alpha < 1$ is required for consumer surplus to be finite, but we have no other restrictions on the choice of this parameter. In future versions we will consider how alternative levels of α will affect the crucial statistics that we study. For the cost function parameter we choose $p = 1$ and $\gamma = 0.1$. This cost function is novel to the literature, and we are still investigating other parameter choices. We set the rate of capital depreciation at 10 percent a year, a standard rate assumed in the real business cycle literature. We set proportional storage costs of 5 percent a year, similar to that in Routledge, Seppi, and Spatt (2000) and Pirrong (2012).

The parameters for the demand shock process in (26) that we use are $k_\epsilon = -0.3$, and $\sigma_\epsilon = 0.2$. The drift parameter governing the speed of mean reversion is the same as that in Carlson, Khokher, and Titman (2007), while we choose a lower volatility, since we scale up the volatility by the amount $(1 + |\epsilon_t|)$.

We assume that the price of risk is $\sigma_m = 0.3$. This is around the standard level used in asset pricing models to justify an aggregate Sharpe ratio of 30 percent on stocks, close to its historical average. Finally, we assume that the price of capital is constant and set equal to one. Essentially this means that the numeraire good can either be consumed or converted for investment one-for-one. In addition, assuming a constant price of capital implies that none of the investment dynamics in the model arise from it.

We also make some choices on the scale of the problem. We assume that $\bar{x} = 50$, and the total reserves of the resource, $R = 10$. These we believe do not affect the results of the paper.

3.2 Results from a Single Simulation

As we will highlight in this section, this model displays capital cycles that helps explain the variation in the relative basis and the other stylized facts. We start by showing a typical sample path of the model's real and financial variables in Figure 5. Even though we simulate the model for 125 years, we only show the simulated variables until 90 percent of the resource is exhausted. In the displayed simulation, this occurs after about 70 years.

The top left panel shows that simulated demand shock process, which fluctuates around 0. The demand process exhibits some persistent booms and busts. For example between years 25 and 35 demand remained quite strong. It was strong and weak for some shorter episodes as well. The top middle panel shows how the extensive margin expands over time. One noteworthy feature is that the speed of extraction slows after x^e crosses 35 (about 70 percent of total reserves). The top right panel shows the optimal consumption of the resource. In the simulation shown, consumption peaks at the time of the first demand spike at about 15 years, and the time of the second major boom from years 25-35, it was only about half the peak value. Subsequently, in years 35 and beyond, consumption is positive, but only rises above a very low level in periods of spiking demand. Therefore, the model appears to support the peak consumption view.

The middle panels show the real decisions made by the firm. The left panel shows the capital accumulation process. This process is strongly (61 percent) correlated with the demand shock process, but has one additional feature. It spikes up faster than it can decline. The spiking up happens due to spikes in investments in periods of strong demand shown in the

middle panel. However, once capital is in place, it can only decline at the pace of depreciation when demand falls off. Given this constraint, the firm optimally invests zero in periods of low demand, and rapidly increases investment once growth resumes. In future versions of the paper, we will impose tighter constraints on investment so that it might follow a less volatile process.

The middle right process shows the inventory process chosen by the firm. As seen, inventory is positive frequently, specially after year 35, which we noted was the time after which extraction slowed. Even prior to this year, production (related to changes in the extensive margin) stops in periods of low demand and inventory is used to sustain consumption in such periods. After year 35, production remains mostly off (about 2/3 of the years) and inventory has a fairly mechanical cycle only disturbed by extremely high demand. Still, the ability to accumulate inventory is critical to maintaining a small level of consumption for a long period of time. We only display until year 70 of the simulation, but it continues at this small level until year 125 (which we simulated) and beyond. Despite the difference in production and investment before and after year 35 and the drop off in consumption, the capital and investment process actually look quite similar. In our model, resources get increasingly costly to extract as the extensive margin expands, requiring increasing amounts of capital for the same level of consumption. Therefore, we still see a cyclical variation in the capital stock similar to earlier part of the simulation. However, the ratio of capital and inventory to consumption trend upwards over time after significant depletion of the resource.

The bottom left panel shows how the spot price of oil looks over time in the model. As seen, the price trends up with occasional fluctuations until year 50, and then drops off a bit and still fluctuates with demand shocks. Comparing this series with consumption (top right panel) we see that the trend in the price is related to the decline in consumption, although the correlation is far from perfect. Indeed prices are also affected by the capital accumulation and inventory processes which are the supply responses of firms to demand shocks and depletion rates.

The middle panel shows the weak relative basis and its fitted value based simply on a linear regression on the capital process. As seen, the two series are indeed positively related, with an R^2 of about 12 percent. We recall from Table 1 that in the data the R^2 was about twice as high,

however, a large part of our data sample has a single episode of a large increase in the capital stock, and the relationship is not as strong in other periods. Why does the basis increase in periods of high capital in our model? In such periods there are lower extraction costs and higher extraction rates, but also greater buildup of inventory, which serves to moderate consumption and hold up spot prices. In fact, since the level of capital is persistent, investors expect continued low extraction costs, and high inventory accumulation to support high prices in the near future, leading to a positive basis.

Finally the bottom right panel shows the risk premium in the model. It has a negative relationship, as in the data, with both capital and inventory, with an R^2 of about 4 percent, comparable to that in the data. This results because in periods of high capital, extraction costs are expected to remain low, leading to less volatile future consumption, and hence a lower risk premium. This is similar to the comparative static result that we found in Section 1.3. In addition, the risk premium tends to be more volatile after significant depletion of the resource as future consumption becomes riskier when inventory depletes.

How does the variance of the simulated spot prices at alternative frequencies compare with that in the data? We look again at Figure 3, where the left and right panels show the frequency decomposition in the data and the simulated model path. As seen, the model plot shows a similar U shaped pattern of the decomposition, with the largest variance coming from very low frequency and very high frequency shocks. Compared to the model though, there is some greater variance from business cycle frequencies, which arises as the spot price fluctuate with demand shocks. Still, the demand process essentially has no very low frequency variation, while the spot prices do. This arises due to the exhaustibility of the resource, which leads to the upward trend in spot prices, similar to that in the data. Moreover, due to the capital accumulation process, the price spikes are not perfectly correlated with demand shocks, which have a strong business cycle component.

3.3 Results from Multiple Simulations

While the results from the single simulation shed light on the observed relations between real and financial variables, due to exhaustibility, we are limited in the length of the simulation. In addition, the exact sequence of demand shocks will determine the rate of exhaustion and

potentially the statistical significance of our results. In this section, we simulation multiple times and examine the long run distribution of several statistics.

The results are shown in Figure 6. The top left panel shows the distribution of the time to reach 90 percent exhaustion of the resource. As can be seen the mean time to exhaustion is about 90 years although there is considerable variation. The top right panel shows the distribution of the year to maximum consumption. As shown in the single simulation, maximum consumption typically occurs at the first time there is a sustained increase in demand. Indeed the histogram of the time of peak consumption has the largest mass in the first two years, but also a high likelihood of a maximum at about 5 years. There are some paths where the peak occurs at about 15 to 20 years. However, in all the paths simulated, we obtained consumption peaks significantly before depletion of even 50 percent of the resource.

The middle right panel looks at the trend of the spot price. As seen, in most paths there is an upward trend in spot prices. We simulated paths of 125 years, and found that the most frequently, the maximum spot price occurred in years 120-125. In all these paths, consumption declined to a very low level after around year 40 to 50. While this may not line up exactly with historical patters where we have observed high consumption for far longer, it is likely we can get slower exhaustion by adjusting the parameters of the demand shock and preferences, which we will do in future versions of the paper.

The middle right panel shows the distribution of the R^2 of the regression of the basis on the capital stock in each sample path. The R^2 varies between 0 and 20 percent, with a mean of 5.7 percent. This is lower than the data R^2 of about 24 percent (see Table 1), but we must recall that in our data sample, there has been a sustained period of rapidly accumulating capital in the 2000s, which the model predicts will cool off when demand weakens. The bottom right panel shows the R^2 of the basis on inventory, and as seen, it varies between 0 and 2.5 percent. This is lower than in the data, which has been seasonally adjusted. We must again find parameters that get this relationship stronger, as in the data.

Finally, the bottom right panel simulated the R^2 of the risk premium on the capital stock and inventory. Here the R^2 average across simulations is about 6.5 percent, which is stronger than in the data as seen in Table 2. Still, the sign of the coefficients agrees with the data, and

points out that the same decision variables are important components of the risk premium in oil futures.

It is finally noteworthy, that the regression results in the model are similar to that in the data despite our explicit modeling of exhaustibility. While peak consumption is a debatable hypothesis, it is useful to note that its implications are not at odds with the data. Indeed, our results suggest that the regression statistics will continue to be significant even if we see rapid declines in consumption. This arises in our model because greater capital and more frequent inventory accumulation are required to sustain the same level of consumption.

4 Conclusion

In this paper, we provide a new model of exhaustible resource extraction, inventory accumulation, and accumulation of E&D capital, to address stylized facts on real and financial variables of the oil industry. The model shows that such evidence is consistent with the peak consumption view and an increasing trend of real oil prices.

The model shows that even though consumption peaks, and real prices of the resource trend upwards, the cycles of inventory and capital accumulation are quite stable, as increasingly costlier resources are extracted. While the resource is not completely exhausted even after more than a hundred years, consumption drops significantly after about half the resource base is exhausted, while capital fluctuates with the demand cycle. Therefore, the positive relationship between E&D capital and the futures basis, and the negative relationship between capital and the risk premium, are consistent with a model of near exhaustion of the resource, or the peak consumption view. The model generates a frequency decomposition of the futures basis with significant non business cycle components and relatedly, an increasing trend in spot prices similar to that in the data. Overall, our model shows that simultaneously modeling the long run (E&D) and short run (inventory) decisions of firms, is important in understanding the relationship between real and financial variables of the oil industry.

Data Appendix

We obtain historical crude oil futures contracts prices from July 1986 to November 2014 from the *Chicago Mercantile Exchange* (CME). The data series provided summarize the prices

from all public traded exchanges. We obtain the series of constant maturity Treasury yields from the *Federal Reserve Board*, which are required for calculating the weak relative basis. We filter the series and use only prices for contracts with positive volume. We obtain the core CPI (to deflate spot oil prices) from the St. Louis Fed.

“Inventory” is denoted as the total US stock of crude oil and petroleum products ex-strategic oil reserves. We obtain these data from the US Energy Information Administration (EIA). “Capital Stock” is the sum of the “Property Plant and Equipment” variable in Compustat of firms in oil and gas field exploration services (SIC code 1382). Consumption of petroleum products is provided by the EIA.

Appendix 1

Proof of Proposition 1.

Using the equilibrium stock price at date 1 in (11), we have that the call option value is simply

$$\begin{aligned}
C(x|x_0^e, K_1) &= e^{-r} E^Q \left[\left(\frac{a e^{\mu+\sigma\epsilon} + \frac{x_0^e}{\bar{x}} R_0}{b + \frac{R_0}{\bar{x}g(K_1)}} - x g(K_1) \right)^+ \right] \\
&= \frac{e^{-r}}{D} E^Q \left[\left(a e^{\mu+\sigma\epsilon} - (D x g(K_1) - \frac{x_0^e}{\bar{x}} R_0) \right)^+ \right] \\
&= \frac{a e^{-r}}{D} \left[E[e^{\mu-\sigma_M\sigma+\sigma\epsilon^*} | \mu - \sigma_M\sigma + \sigma\epsilon^* > \log(k)] - k \text{Prob}[\mu - \sigma_M\sigma + \sigma\epsilon^* > \log(k)] \right] \\
&= \frac{a e^{-r}}{D} \left[e^{(\mu-\sigma_M\sigma+0.5\sigma^2)} N(-d_1) - k N(-d_2) \right],
\end{aligned}$$

as stated. We note that in the third line we use the definition of the ‘risk-neutral shock’ $\epsilon^* = \epsilon + \sigma_M$, while in the fourth line we use the conditional expectation for log normal variables (see e.g. Proposition 2.29 in Nielsen (1999)). The proof for the put is similar. ■

Appendix 2

We proceed by formulating an ‘approximate’ solution to the Hamilton-Jacobi-Bellman equation in 32 using projection methods (Judd 1999, Chapter 11). The value function is denoted as $J(x_t^e, Z_t, K_t, \epsilon_t)$.

STEP 1. Choice of individual basis functions. I choose the Chebyshev polynomials in each of the 4 dimensions: The Chebyshev polynomials on $[-1, 1]$ for the basis for each dimension are

given by

$$q_m(x) = \cos(m \cos^{-1}x),$$

for $m = 1, 2, \dots$, which satisfy the recursive scheme

$$q_{m+1}(x) = 2xq_m(x) - q_{m-1}(x). \quad (41)$$

These polynomials are restricted for the interval $[a, b]$ using the transformation

$$p_m(x) = \frac{q_m\left(\frac{2x-a-b}{b-a}\right)}{\|q_m\left(\frac{2x-a-b}{b-a}\right)\|}.$$

We solve the value function on bounded spaces in each dimension: $[0, \bar{x}] \times [0, \bar{Z}] \times [0, \bar{K}] \times [-\bar{\epsilon}, \bar{\epsilon}]$. The family $\{p_m(x)\}_{m=1,2,\dots}$ are orthonormal polynomials over the chosen intervals.

STEP 2. Choose a basis of ‘complete’ polynomials over the space

The basis of degree M over the 4 dimensions is given by

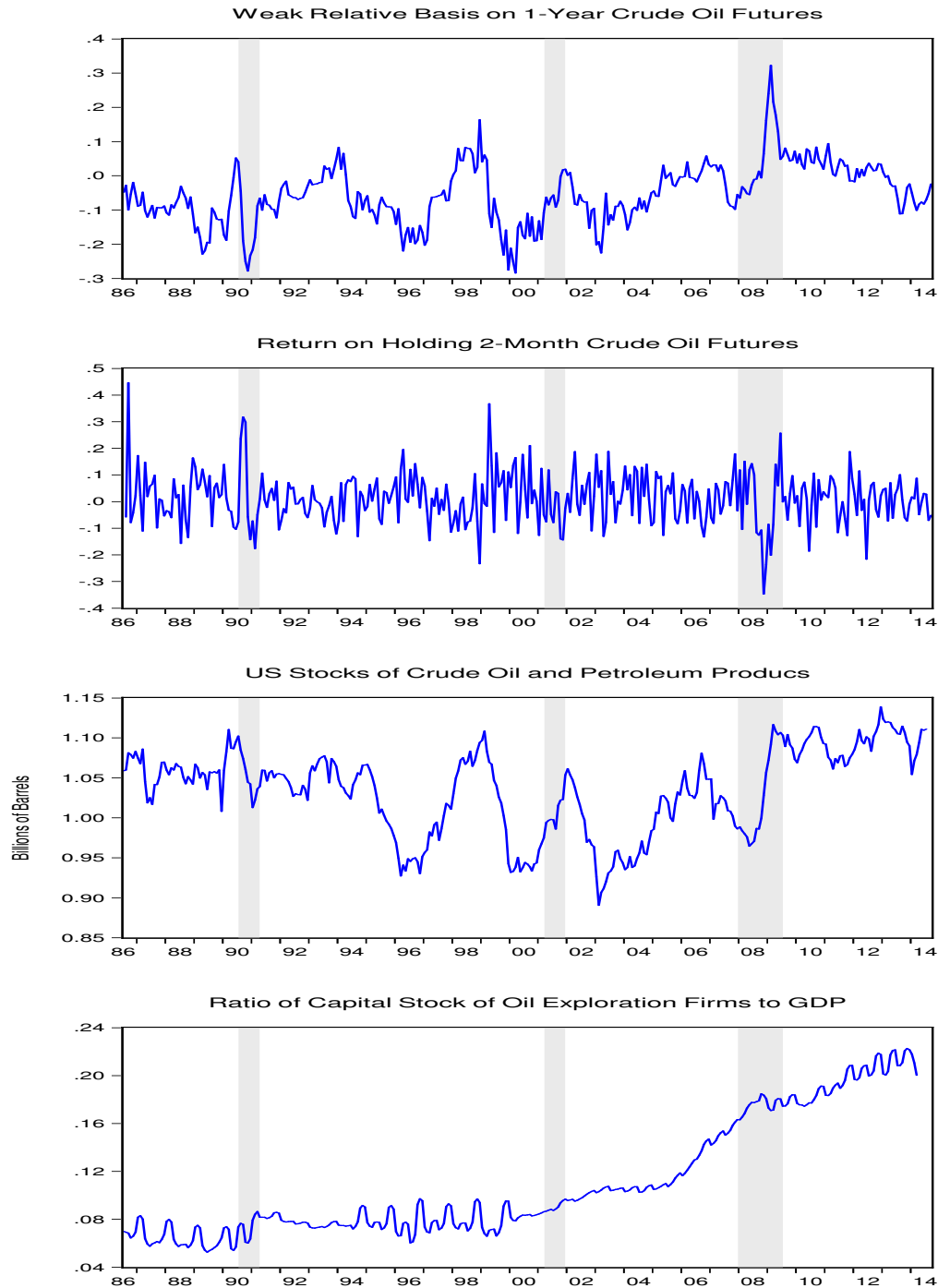
$$\mathcal{P}_M = \{p_{1,i_1}(X) \cdot p_{2,i_2}(Z) \cdot p_{3,i_3}(K) \cdot p_{3,i_3}(\epsilon) \mid \sum_{n=1}^4 i_n \leq M, 0 \leq i_1, \dots, i_3\}$$

We write the generic element of \mathcal{P}_M as $\phi_m(X, Z, K, \epsilon)$, $m = 1, 2, \dots, M^c$, where M^c is the length of the complete polynomial basis. The set of complete polynomials for a 4 dimensional problem grows polynomially in 4, as opposed to the tensor product basis which would use every possible product of the degree- M individual basis functions, and hence would grow at the rate of M^4 (see, e.g., pp. 239 in Judd 1999). The complete polynomials asymptotically, as M becomes large, provide as good an approximation as the tensor product, but with far fewer elements. . Extending the L^2 norm over the 4-dimensional space as the 4-fold integral, it can be verified that the basis of complete polynomials is orthonormal on the bounded Cartesian product space.

STEP 3 Let $b^{(n)}$ be the n th guess on the coefficients of the polynomial, i.e $J^{(n)}(x_t^e, Z_t, K_t, \epsilon_t) = \sum_{m=1}^{M^c} b^{(n)} \cdot \phi_m(X, Z, K, \epsilon)$. Then we solve 32 for the $n+1$ th guess as $J^{(n+1)}(x_t^e, Z_t, K_t, \epsilon_t)$, using the first order conditions (33) – (37). Note that we are able to take partial derivatives of the n th guess value function, which is just a polynomial sum. The first order conditions are solved on a discrete grid of values for inventory, investment, and the extensive margin.

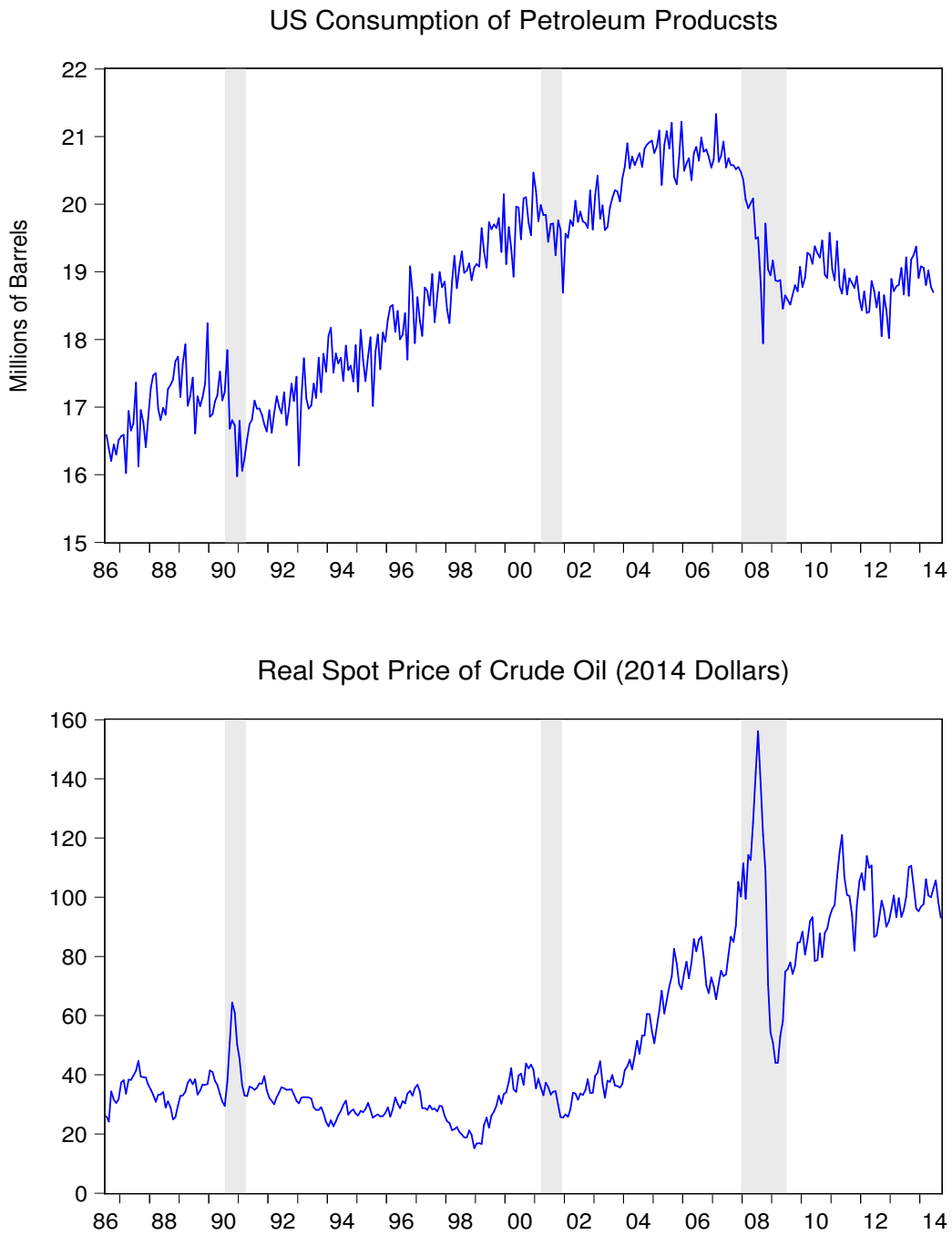
STEP 4 We now appeal to the Chebyshev Interpolation Theorem (see Judd (1999)) to find an approximate solution to the Bellman equation. Denote $Y = (x^e, Z, K, \epsilon)$. The approximation is made by evaluating the $J^{(n+1)}(Y)$ at the Chebyshev zeros in the Cartesian space, given the coefficients $b^{(n)}$. Each interpolation point therefore provides us a linear equation in the coefficients $(b_m)^{(n+1)}$. With M^I interpolation points, we have an overidentified system of equations in M^c unknown coefficients, and we solve for $(b_m)^{(n+1)}$ using linear regression. We then repeat until convergence.

Figure 1: Crude Oil Futures Basis and Returns, Oil Inventory and Capital Stocks of Oil Exploration Firms (1986:7-2014:9)



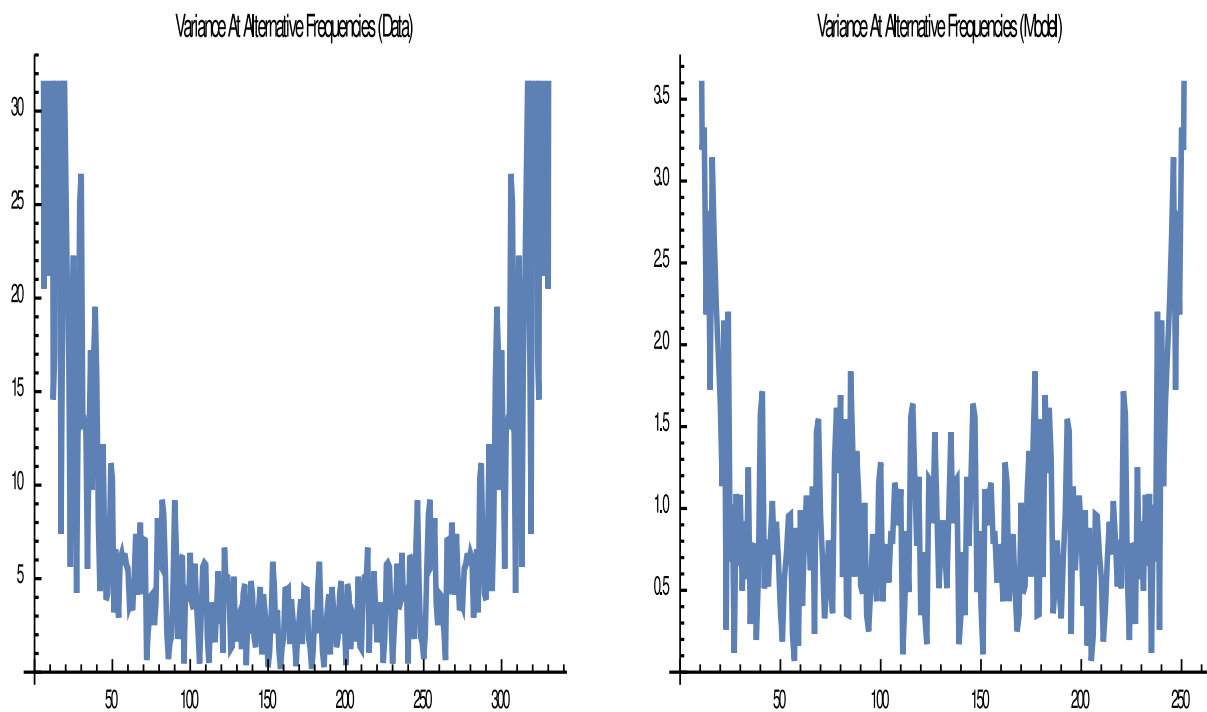
The **top panel** shows the seasonally adjusted weak relative basis on 1-year crude oil futures contracts, which in month t is $[e^{-r(t)}F(t) - S(t)]/S(t)$, where $F(t)$ is the 1-year futures prices at the beginning of each quarter and $S(t)$ is the spot price of WTI oil in Cushing, Oklahoma. Seasonal adjustment is done using the X-12 procedure (used by the US Department of Commerce). The **second panel** reports the return from month $t - 1$ to month t of a 2-months WTI futures contract. The **third panel** shows the seasonally adjusted “Inventory” of total US stock of crude oil and petroleum products (in billions of barrels) excluding special purpose reserves at the end of each month. The **bottom panel** shows the “Capital Stock” which is the sum of the “Property Plant and Equipment”

Figure 2: Consumption of Petroleum Products and Real Spot Price of Crude Oil (1986:7-2014:6)



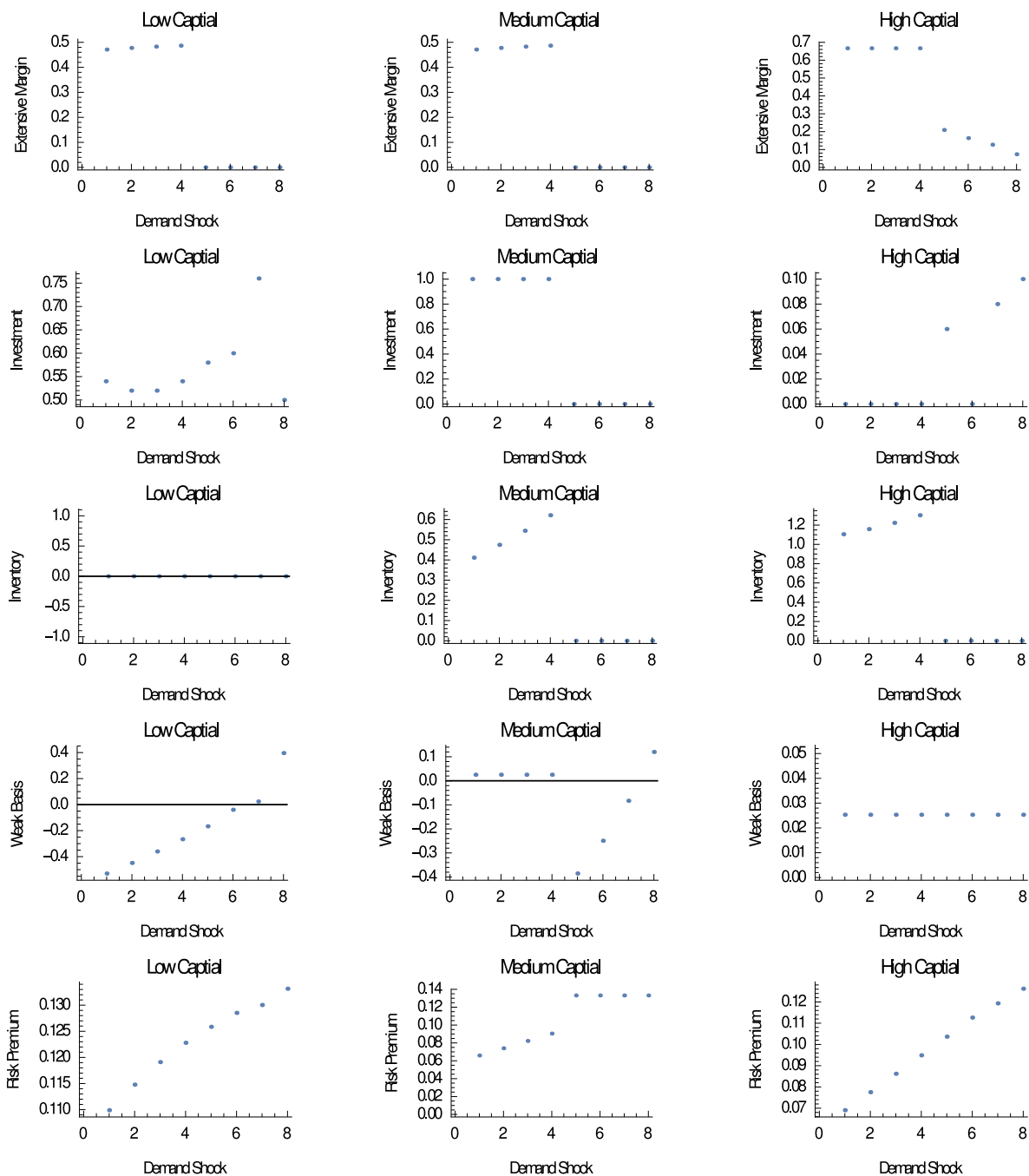
US consumption of petroleum products is reported by the EIA. The spot price is the price of a futures contract with less than one month to maturity reported at the beginning of the month. The real spot price is the spot price divided by core CPI (excluding food and energy). 30

Figure 3: Variance Frequency Decomposition of the Price of Oil



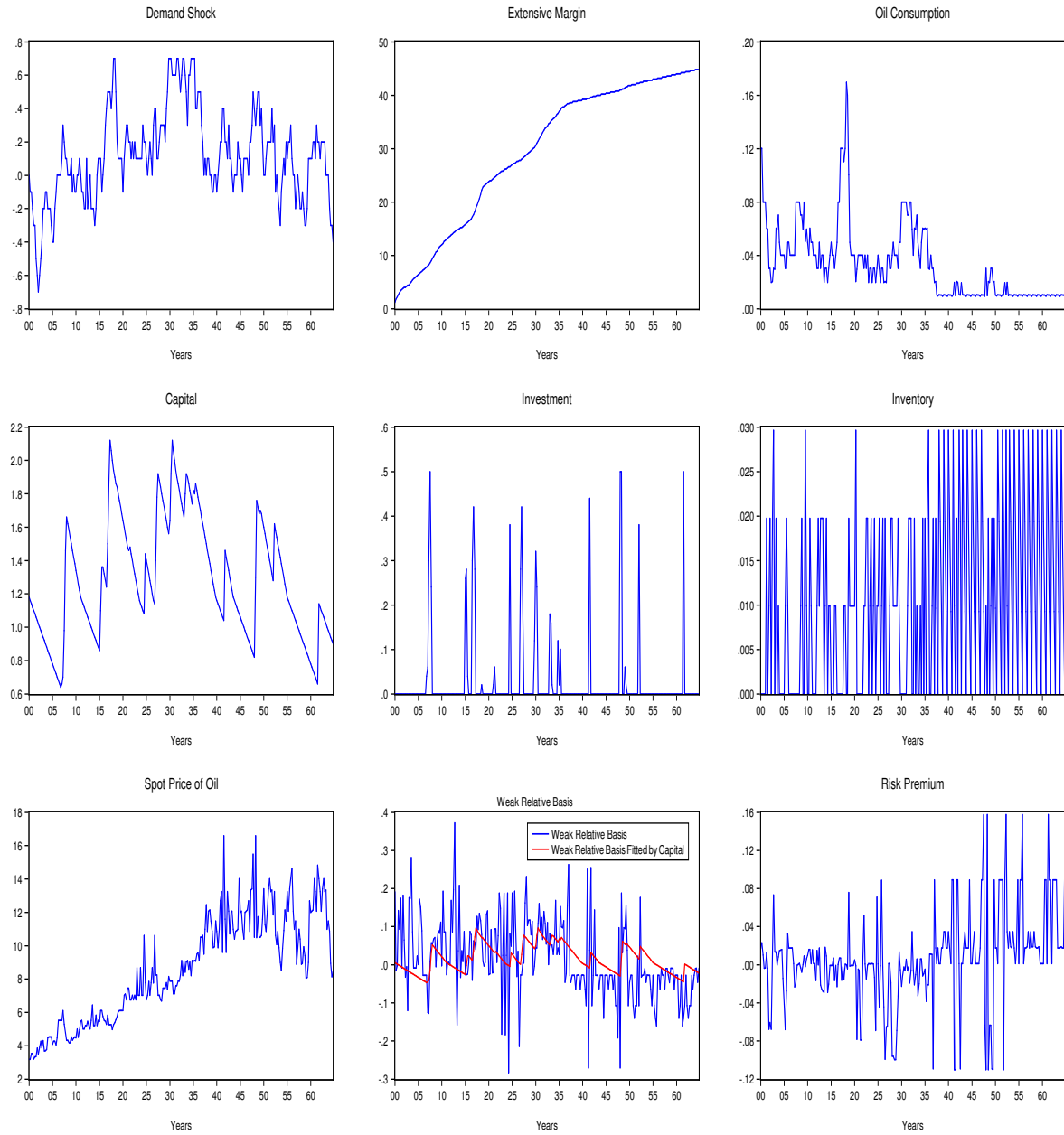
We report the variance frequency decomposition (Fourier Transform or spectrum) of the real WTI spot price from 1986:7 to 2014:9 (left panel) and from simulated spot prices from our infinite horizon model in Section 2 (right panel).

Figure 4: Comparative Statics of Optimal Firm's Decisions, Futures Basis, and the Risk Premium with Respect to the Mean Demand Shock in the 2-Period Model with a Linear Demand Function



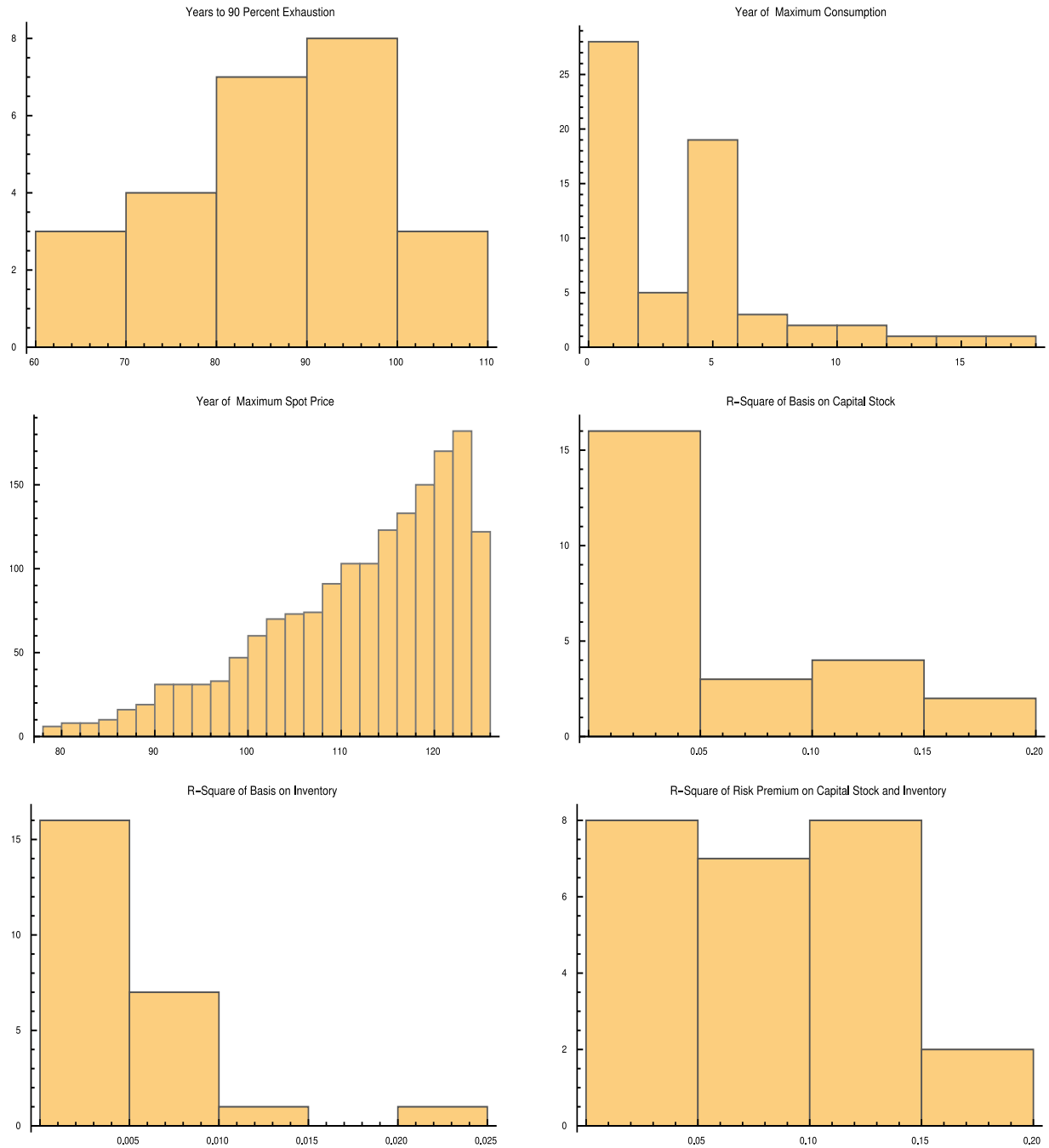
We report the comparative statics of inventory, investment, the weak relative basis, and the risk premium in the 2-period model with respect to alternative levels of the demand shock. The parameters of the model are $r = 0.02$, $a = 2$, $b = 1$, $R = 3$, $Z_0 = 0.5$, $\bar{x} = 1$, $\sigma = 0.5$, $\gamma_0 = 0.2$, $\gamma_1 = 0.2$, $\delta = 0.1$, $\sigma_m = 0.25$, $u = 0.025$. 'Low', 'Medium', and 'High' capital stock levels are $K = 0.5$, $K = 1$, and $K = 1.5$, respectively. The demand shock is assumed to vary between -0.7 and 0.7 in increments of 0.2 .

Figure 5: Optimal Firm's Decisions, Futures Basis, and the Risk Premium from a Single Simulation of Infinite Horizon Model



We report the simulated time series of the model in Section 2. The parameter choices are in Section 3.1.

Figure 6: Distribution of Exhaustion Times, Consumption, Spot Prices, and Financial Variable Regressions from Multiple Simulations.



We report the simulated time series of the model in Section 2. The parameter choices are in Section 3.1.

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